Coulomb Effects on Electromagnetic Pair Production in Ultrarelativistic Heavy-Ion Collisions

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Abstract

We discuss the implications of the eikonal amplitude on the pair production probability in ultrarelativistic heavy-ion transits. In this context the Weizsäcker-Williams method is shown to be exact in the ultrarelativistic limit, irrespective of the produced particles' mass. A new equivalent single-photon distribution is derived which correctly accounts for the Coulomb distortions. As an immediate application, consequences for unitarity violation in photo-dissociation processes in peripheral heavy-ion encounters are discussed.

1 Introduction

The S matrix describing electron scattering at ultrarelativistic pointlike charges was shown to be determined by the gauge phase leading to the Dirac equation represented in the temporal gauge [1]. We found that it naturally exhibits the same form as the well known eikonal expression, as is expected by Lorentz invariance.

The gauge phase leading to the temporal gauge reads

$$\phi(x) = e^{-i\int_{-\infty}^{t} A_0(x)dt'}$$

The consideration of asymptotic states corresponds to sending the upper bound of the integral to infinity. The infinite time integral over the scalar part of the electromagnetic potential in the exponent has to be understood as the principal value of the integral. It can be decomposed into a finite term and an infinite quantity, expressing the familiar divergence of phases in Coulomb scattering. The infinite term can be removed by a gauge transformation which converts the Coulomb boundary conditions of the original problem into a modified short-range potential allowing for asymptotic plane wave solutions.

The ultrarelativistic limit of the gauge transformed potential $A'_0(x)$ reads [2, 3, 4, 1]

$$\lim_{\gamma\to\infty} A'_0(x) = Z\alpha\delta(z-t)\ln x_\perp^2 \tag{1}$$

and hence we obtain for the S operator in coordinate space ($\hat{\gamma}_- = \hat{\gamma}_0 - \hat{\gamma}_3$ is the Dirac matrix structure of the interaction)

$$S = e^{-iZ\alpha\ln x_\perp^2}\hat{\gamma}_- \tag{2}$$

The obtained S operator is a unitary operatorb due to its conformity to the gauge phase. It agrees with the first term of the Magnus expansion of the time-evolution-operator [5], since the considered gauge-transformed interaction was assumed to be compressed to infinitely short times.

This result was proven to be of completely perturbative nature [1]. Note, however, that the perturbative derivation did not require the explicit Fourier transform of the transverse part of (1), which is an ill defined object. For that reason the deduction of the small-coupling limit ($Z\alpha \to 0$) of (2) in momentum space must be treated with special care.

In a rigorous distributional sense it can be defined as the *weak limit* $\lambda \to 0$ [6]

$$\int d^2x_\perp e^{-i\vec{k}_\perp \vec{x}_\perp} \ln x_\perp^2 = \lim_{\lambda \to 0} 4\pi \left(\frac{-1}{k_\perp^2 + \lambda^2} - \pi \delta^2(k_\perp) \ln \left(\frac{\lambda^2}{\mu^2} \right) \right) \tag{3}$$

with $\lambda = \omega/\gamma$, $\mu = 2/e^C$. 'Weak limit' means that the limit $\lambda \to 0$ has to be taken after having integrated the expression with a test function. The second term on the RHS arises from the gauge transformation applied to the potential and thus accounts for the Coulomb distortions.

Discarding the second term and taking the limit $\lambda \to 0$ directly would accidentally yield an expression for the Fourier transform of (1), being identical to the Fourier transform of the ungauged potential in the limit $\gamma \to \infty$

$$\int d^4x \; e^{ikx} A_0 = -(2\pi)^2 Z\alpha \delta(k_-) \frac{2}{k_\perp^2}$$

This error is made if one intends to extract the correct small-coupling limit from a naive Taylor expansion of the Fourier transformed T matrix whose linear term reads

$$\lim_{Z\alpha \to 0} T(k) \approx (2\pi)^2 \delta(k_-) i Z\alpha \frac{2}{k_\perp^2} \overline{u}(p') \hat{\gamma}_- u(p) \tag{4}$$

Here $u(p)$ is the electron unit spinor, p and p' are the initial and final momenta of the electron, repectively, $k = p' - p$.

Since, however, Taylor expansion and Fourier transformation do not commute in this case, the Taylor expansion of the Fourier transformed T matrix for this purpose is not justified.[1] The correct small-coupling limit of the scattering amplitude in momentum space can thus not be found by a naive Taylor expansion of the Fourier transformed T matrix and does not agree with first-order perturbation theory. According to (3) this is simply based on the fact, that the gauge transformed potential correctly accounts for Coulomb boundary conditions.

In the following we want to investigate, how the correct treatment of Coulomb boundary conditions in all orders of perturbation theory influences the cross section of the scattering process.

[1] Note, that with the above mentioned distribution-theoretical precautions, it is possible to obtain the correct result via Taylor expansion [7].

2 Implications on the cross section

We consider the exact amplitude for electron scattering at an ultrarelativistic pointlike charge, moving in $+z$ direction. We state it in terms of the invariant squared momentum transfer $t \approx -k_\perp^2$ for $\gamma \to \infty$[2]

$$
\begin{aligned}
A &= 2\pi\delta(k_-)F_{p',p}(e^{-iZ\alpha\ln x_\perp^2}-1)\overline{u}(p')\hat{\gamma}_-u(p) \\
&= -i8\pi^2 Z\alpha\delta(k_-)\frac{1}{t}\frac{\Gamma(-i\alpha Z)}{\Gamma(i\alpha Z)}e^{-iZ\alpha\ln(-t/4)}\overline{u}(p')\hat{\gamma}_-u(p) \qquad (5)
\end{aligned}
$$

$F_{p',p}$ abbreviates the Fourier transform with respect to the transverse coordinates, taken at the momentum $\vec{k}_\perp = (\vec{p'}_\perp - \vec{p}_\perp)$. The cross section for this scattering process is found to be exactly the Mott formula for Coulomb scattering of ultrarelativistic electrons at a static source, Lorentz-transformed into the electron's rest frame. Such kind of agreement between the exact result and the first order perturbation theory is also found in the nonrelativistic case, known as one of the peculiarities of the Coulomb field.

The well established eikonalization of the scattering amplitude and thus the reduction to Mott's result imply, that in the high-energy limit the electron and the positron Coulomb scattering cross section become identical. This behaviour of the cross section at ultrarelativistic energies is required by the Pomeranchuk theorem [8].

One can draw an analogy to pomeron exchange in hadron physics: The scattering process can be described in terms of the single exchange of an "effective photon" according to the modified potential

$$
V_0(x) = V_3(x) = \delta(z-t)\left(\left(\frac{1}{x_\perp}\right)^{2iZ\alpha} - 1\right) \qquad (6)
$$

In the field of two ultrarelativistic colliding pointlike nuclei, the exact scattering amplitude was shown to retain the structure of the second-order perturbative result, due to the causal decoupling property [1]. Each interaction can be described by the modified potential (6). In the following we consider the symmetric collision of two ions with charge Ze, the extention to asymmetric collisions is trivial.

Accounting for both time orderings, the amplitude reads

$$
\begin{aligned}
A^{tot}_{p'p} &= \int \frac{d^2k_\perp}{(2\pi)^2} F_{k,p}(e^{-iZ\alpha\ln x_\perp^2}-1)F_{p',k}(e^{-iZ\alpha\ln x_\perp^2}-1)e^{i(\vec{p'}_\perp-\vec{k}_\perp)\cdot\vec{b}} \\
&\Bigg(\overline{u}(p')\frac{-\hat{\vec{\alpha}}_\perp\cdot\vec{k}_\perp+\gamma_0 m}{p'_+p_- - k_\perp^2 - m^2 + i\epsilon}\hat{\gamma}_+u(p) \\
&\overline{u}(p')\frac{-\hat{\vec{\alpha}}_\perp\cdot(\vec{p}_\perp+\vec{p'}_\perp-\vec{k}_\perp)+\gamma_0 m}{p'_-p_+ - (\vec{p}_\perp+\vec{p'}_\perp-k_\perp)^2 - m^2 + i\epsilon}\hat{\gamma}_-u(p)\Bigg) \qquad (7)
\end{aligned}
$$

[2]Note the striking similarity between (5) and the nonrelativistic (Schrödinger) amplitude

$$
f(\theta) = -\frac{1}{2k^2\sin^2\frac{\theta}{2}}\frac{\Gamma\left(1+\frac{i}{k}\right)}{\Gamma\left(1-\frac{i}{k}\right)}e^{-\frac{i}{k}\ln\sin^2\frac{\theta}{2}}
$$

with the squared momentum transfer being proportional to $\sin^2\theta/2$.

Here the trajectory of one ion is shifted by the impact parameter $\vec{b}$, which is accounted for by the factor $e^{i(\vec{p'}_\perp - \vec{k}_\perp)\cdot\vec{b}}$.

We now use the crossing invariance of the amplitude to apply the obtained result to electron-positron pair production. The initial electron four momentum p has then to be replaced by the negative final positron momentum $p \to -p^p$. The final electron momentum will be denoted by $p' = p^e$. With (7) we obtain for the pair production probability

$$
\begin{aligned}
\frac{d\sigma}{d^2b} &= |A^{tot}_{p'p}|^2 \frac{md^3p^e}{(2\pi)^3E^e}\frac{md^3p^p}{(2\pi)^3E^p} \\
&= \frac{md^3p^e}{(2\pi)^3E^e}\frac{md^3p^p}{(2\pi)^3E^p}\int\frac{d^2k_\perp}{(2\pi)^2}\int\frac{d^2k'_\perp}{(2\pi)^2}F_{k,-p^p}(e^{-iZ\alpha\ln x_\perp^2}-1)F_{p^e,k}(e^{-iZ\alpha\ln x_\perp^2}-1) \\
& F^*_{k',-p^p}(e^{-iZ\alpha\ln x_\perp^2}-1)F^*_{p^e,k'}(e^{-iZ\alpha\ln x_\perp^2}-1)e^{i(\vec{k'}_\perp-\vec{k}_\perp)\cdot\vec{b}} \\
& \left(\overline{u}(p^e)\frac{-\hat{\vec{\alpha}}_\perp\cdot\vec{k}_\perp+\gamma_0 m}{-p^e_+p^p_- - k_\perp^2 - m^2 + i\epsilon}\hat{\gamma}_+ u(-p^p)\right. \\
& \left.+\overline{u}(p^e)\frac{-\hat{\vec{\alpha}}_\perp\cdot(-\vec{p^p}_\perp+\vec{p^e}_\perp-\vec{k}_\perp)+\gamma_0 m}{-p^e_-p^p_+ - (-\vec{p^p}_\perp+\vec{p^e}_\perp-k_\perp)^2 - m^2 + i\epsilon}\hat{\gamma}_- u(-p^p)\right) \\
& \times\left(\overline{u}(p^e)\frac{-\hat{\vec{\alpha}}_\perp\cdot\vec{k'}_\perp+\gamma_0 m}{-p^e_+p^p_- - {k'}_\perp^2 - m^2 + i\epsilon}\hat{\gamma}_+ u(-p^p)\right. \\
& \left.+\overline{u}(p^e)\frac{-\hat{\vec{\alpha}}_\perp\cdot(-\vec{p^p}_\perp+\vec{p^e}_\perp-\vec{k'}_\perp)+\gamma_0 m}{-p^e_-p^p_+ - (-\vec{p^p}_\perp+\vec{p^e}_\perp-k'_\perp)^2 - m^2 + i\epsilon}\hat{\gamma}_- u(-p^p)\right)^* \qquad (8)
\end{aligned}
$$

The integration over the impact parameter yields the pair production cross section. Due to the $\delta^2(\vec{k'}_\perp - \vec{k}_\perp)$-function occuring in the $\vec{b}$ integration, a further momentum integral can be perfomed, leaving

$$
\begin{aligned}
d\sigma &= \frac{md^3p^e}{(2\pi)^3E^e}\frac{md^3p^p}{(2\pi)^3E^p}\int\frac{d^2k_\perp}{(2\pi)^2}|F_{k,-p^p}(e^{-iZ\alpha\ln x_\perp^2}-1)|^2|F_{p^e,k}(e^{-iZ\alpha\ln x_\perp^2}-1)|^2 \\
& \left|\overline{u}(p^e)\frac{-\hat{\vec{\alpha}}_\perp\cdot\vec{k}_\perp+\gamma_0 m}{-p^e_+p^p_- - k_\perp^2 - m^2 + i\epsilon}\hat{\gamma}_+ u(-p^p)\right. \\
& \left.+\overline{u}(p^e)\frac{-\hat{\vec{\alpha}}_\perp\cdot(-\vec{p^p}_\perp+\vec{p^e}_\perp-\vec{k}_\perp)+\gamma_0 m}{-p^e_-p^p_+ - (-\vec{p^p}_\perp+\vec{p^e}_\perp-k_\perp)^2 - m^2 + i\epsilon}\hat{\gamma}_- u(-p^p)\right|^2 \qquad (9)
\end{aligned}
$$

Thus, upon integration over the whole impact parameter plane, the phases in the individual scattering amplitudes (see (5)) cancel. Consequently, in the limit $\gamma \to \infty$ the cross section is found to reduce to the lowest-order perturbation theory, the two-photon result. This behaviour does not only naturally explain [10] the experimentally observed quadratic dependence on the target's and the projectile's charge [9], but also implies, that no asymmetries should occur in electron and positron spectra.

Equation (9) is strictly valid only for pointlike ions. The focus on electromagnetic reactions in peripheral heavy-ion collisions implies a restricted range of impact parameters with a lower bound $b = r_A + r_B$, r_A and r_B being the radii of the ions. Therefore in

experiments which are triggered on peripheral collisions, effects of the Coulomb distortions described by the phase in (5) will be visible.

The eikonal approximation (and thus the cross section) is known to become energy-independent in the ultrarelativistic limit [11]. This dependence is restored by accounting for the correct transverse momentum range, which is restricted by the validity of (1). This condition reads [12, 1]

$$|\vec{k}_\perp| \gg \frac{\omega}{\gamma} \tag{10}$$

Such a low energy cut off is also necessary to cure the divergence in (7).

3 Equivalent Photon Approximation

We intend to study the behaviour of the cross section, both impact parameter dependent and impact parameter integrated, in the Weizsäcker-Williams method of equivalent photons. This approximation uses the similarity between the fields of a fast moving charge and a swarm of real photons moving in beam direction. It approximately correstponds to the first order Born approximation in the temporal gauge: Only the transverse part of the interaction is considered – the longitudinal part is suppressed by $1/\gamma^2$ – and the vertex function is evaluated on-shell at $k^2 = 0$, i.e. for an assumed real photon. Rewriting the exact cross section in terms of the real photon cross section, the whole information about the scattering potential, which can be the retarded Coulomb potential or the modified potential (6), respectively, is then contained in the distribution function of the equivalent photons $n(\omega)$. Roughly speaking, this photon distribution function is determined by the squared absolut value of the Fourier-transformed potential (in temporal gauge). The obvious advantage of casting the exchange of effective photons according to (6) in the Weizsäcker-Williams form is, that any difference between the second-order perturbative result and the exact calculation will be solely generated by differences between the equivalent photon distributions.

The Weizsäcker-Williams approximation is applicable if the exchanged momentum meets the following conditions [13]

$$\frac{\omega}{\gamma^2} \ll |\vec{k}_\perp| \ll m \tag{11}$$

and

$$|\vec{k}_\perp| \ll \omega \ll m\gamma \tag{12}$$

The upper bounds mainly stem from the requirement, that $|k^2|$ is negligible compared to $2p \cdot k \geq m^2$, such that the intermediate propagators of the scattered particles can be approximated by those describing the interaction with real photons. The particle's rest mass in (11) is however a conservative upper bound and the equivalent photon method is not strictly invalid for $|k^2| \sim m^2$. Note, that for the approximative calculations in [14], the transverse mass of the scattered particle was taken as the upper bound for (11).

Since the exact amplitude takes the eikonal form, we must point out the following: The expansion of the ultrarelativistic scattering amplitude in powers of the transferred momentum yields, as the leading term, the eikonal expression (describing the minimal deflection from the initial straight-line trajectory) [15, 16]. Its perturbation-theoretical

derivation requires that the quadratic terms k^2 are negligible relative to the terms $2p_i \cdot k$ in the denominators of the propagators, where k is any partial sum of the internal momenta [17]. The exact validity of the eikonal formula at infinite energies therefore shows, that the transferred momentum $|k^2|$ does not exceed m^2 (irrespective of the value of m). This agrees with the theoretical observation, that at high energies particles are predominantly scattered into a cone with opening angle $\theta \sim 1/\gamma$, corresponding to momentum transfers $|k^2| \sim m^2$. The main contributions to the cross section are thus expected from spatial distances larger or equal the Compton wavelength of the particle.[3]

Moreover, the longitudinal part of the interaction vanishes identically in the limit $\gamma \to \infty$. Hence, the applicability conditions of the Weizsäcker-Williams approximation are trivially fulfilled in the limit $\gamma \to \infty$.[4]

We have the freedom to apply this method to the interaction of the electron with both nuclei, giving two possibilities (see Figure 1)

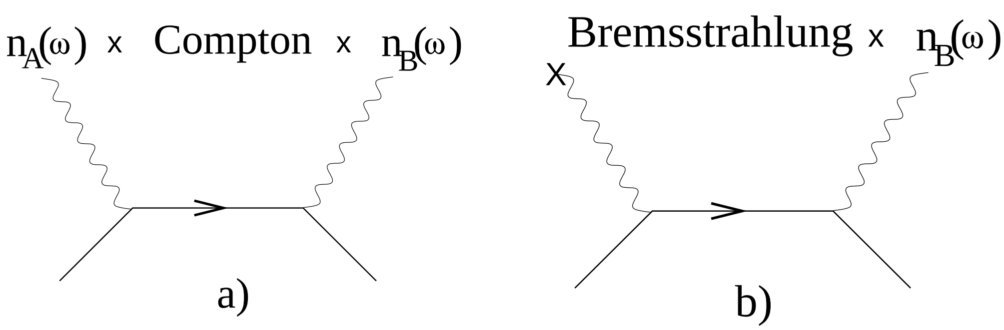

Figure 1: The two possible distinct processes, that can be used to describe electron-positron production in heavy-ion collision. One or both ultrarelativistic ions can be replaced by an equivalent photon distribution. If the bremsstrahlung process b) is calculated in one ion's rest frame, the electron must be assumed ultrarelativistic, to yield agreement with a).

The two possible calculation schemes (Figure 1) agree, since in b) the bremsstrahlung emission and the scattering at the external potential decouple. This is due to the fact, that the region in which the ultrarelativistic electron "feels" the external field is assumed to be pointlike and any frequency of the emitted photon is "soft" compared to the timescale of the scattering. Coulomb effects arise, if one explicitly accounts for the finite interaction time, either in the scattering process by correcting the eikonal formula or by keeping the eikonal amplitude for the scattering process but assuming a Rutherford-deflected trajectory for the photon emission [13]. Corrections to the eikonal formula account for higher orders of e.g. the Magnus expansion [5], which is an expansion in the interaction time τ around the instantaneous interaction $\tau \sim 1/\gamma \to 0$. In general these Coulomb

[3]From the asymmetry of electron and positron spectra produced in S(200 GeV/n)+Au collisions , the mean transverse distance from the target ion was deduced to be approximately two Compton wavelengths [9]. The collision energy corresponds to $\gamma \approx 10$ in the center of speed system.

[4]Just as the eikonal formula, the Weizsäcker-Williams approximation can be viewed as the leading term of an expansion in powers of k^2/m^2 [18]. The validity of the eikonal expression then automatically implies the validity of the Weizsäcker-Williams method.

effects vanish, if the energy of the emitted photon is too small to resolve details of the scattering process, and the recoil of the electron is negligible.

To apply the Weizsäcker-Williams method to the bremsstrahlung photon, the recoil of the bremsstrahlung photon must, however, be assumed negligible. The small momentum transfer is in turn ensured by the eikonalization of the scattering process.

The equivalent single-photon distributions $n_{A/B}(\omega)$ of the ions A and B, are determined from the effective potential (6). The photon distribution reads

$$n(\omega) = \frac{1}{4\pi^2\alpha\omega}\int_{\omega/\gamma}^{m} k_\perp dk_\perp \left| k_\perp \pi Z\alpha \left(\frac{4}{k_\perp^2}\right)^{1-iZ\alpha} \frac{\Gamma(-i\alpha Z)}{\Gamma(i\alpha Z)}\right|^2 = \frac{2Z^2\alpha}{\pi\omega}\ln\left(\frac{m\gamma}{\omega}\right) \tag{13}$$

The lower bound of the integral is taken from the condition (10). The upper bound, the electron rest mass, is imposed by (11). The prefactor arises from properly rewriting the cross section (9) in terms of the real photon cross section (i.e. the Compton cross section) and photon distribution functions.

This photon distribution coincides with the equivalent-photon distribution obtained from the Coulomb potential to logarithmic accuracy [13] and is not changed by Coulomb effects.

4 Impact parameter dependent cross section

The impact parameter dependent equivalent photon method for the exact calculation, using the modified potential (6), can be derived similarly to [19]. To this end we have to modify the integrands in (8) such that they account for the limited momentum range ((10) and (11)).

The cut off of low transverse momenta according to (10) can be achieved by the following replacement in (5)[5]

$$t = -k_\perp^2 \rightarrow -\frac{\omega^2}{\gamma^2} - k_\perp^2 \tag{14}$$

This substitution suppresses small transverse momenta less strongly than the strict cut off at $k_\perp = \omega/\gamma$. It assumes the sufficient accuracy of the classical eikonal amplitude for $1/\gamma$ in the near vicinity of $1/\gamma = 0$, which is guaranteed by the possibility of continuous extentions of the eikonal formula towards finite γ and large t. A Yukawa-type damping of transverse distances corresponding to the cut off transverse momenta yields additional terms that change the character of the amplitude significantly and can not be motivated physically.

In accordance with the exact validity of the eikonal formula, the physical situation is such, that the transferred momenta are restricted by the condition $|k^2| \ll m^2$. They are, however, naturally cut off, if one introduces a form factor to account for the finite extent of the nuclei. Thus, large momenta have to be cut off at $k_\perp \approx 1/r_>$, where $r_>$ is the larger value of either the nuclear radius or the Compton wavelength of the scattered particle [21]. In this respect, the electron is an exception, since all other particles have Compton

[5]Note, that for the Schrödinger case the exact validity of the eikonal formula can be proven for a certain off shell domain of the momentum transfer for the whole energy plane [20].

wavelengths smaller or comparable to the nuclear size. To present the calculations in a uniform manner, we use the form factor of the nucleus to cut off the large momenta.

The impact parameter dependent cross section for particle production, described in the equivalent photon method reads [19]

$$\frac{d\sigma}{d^2b} = \int d\omega_1 \int d\omega_2 \left[n_{\|}(\omega_1,\omega_2,\vec{b})\sigma_{\|}^{\gamma\gamma}(\omega_1,\omega_2) + n_{\perp}(\omega_1,\omega_2,\vec{b})\sigma_{\perp}^{\gamma\gamma}(\omega_1,\omega_2)\right] \tag{15}$$

with the two-photon distribution functions $n_{\|/\perp}(\omega_1,\omega_2,\vec{b})$. The elementary two-photon cross sections and the two-photon distribution functions explicitly account for the parallel or orthogonal orientation of the photon-polarizations, denoted by the indices $\|$ and $\perp$, respectively. Since the integration over the impact parameter plane implies an averaging over the photon polarizations, the explicit occurrence of the photon polarizations in the impact parameter dependent cross section is expected. The functions $n_{\|/\perp}(\omega_1,\omega_2,\vec{b})$ can be expressed in terms of single-photon distribution functions $n(\omega,b)$, depending on the transverse separation:

$$n_{\|}(\omega_1,\omega_2,\vec{b}) = \int d^2x_\perp n(\omega_1,\vec{x}_\perp-\vec{b})\, n(\omega_2,\vec{x}_\perp) \left(\frac{(\vec{x}_\perp-\vec{b})\cdot\vec{x}_\perp}{|\vec{x}_\perp-\vec{b}||\vec{x}_\perp|}\right) \tag{16}$$

$$n_{\perp}(\omega_1,\omega_2,\vec{b}) = \int d^2x_\perp n(\omega_1,\vec{x}_\perp-\vec{b})\, n(\omega_2,\vec{x}_\perp) \left(\frac{(\vec{x}_\perp-\vec{b})\times\vec{x}_\perp}{|\vec{x}_\perp-\vec{b}||\vec{x}_\perp|}\right) \tag{17}$$

with

$$n(\omega,b) = \frac{Z^2\alpha}{\pi^2\omega}\left|\int_0^\infty dk_\perp k_\perp^2 \frac{F(k_\perp^2+\omega^2/\gamma^2)}{(k_\perp^2+\omega^2/\gamma^2)^{1-iZ\alpha}} J_1(bk_\perp)\right|^2 \tag{18}$$

J_1 is a Bessel function. The function F denotes the chosen form factor of the nucleus.

For a pointlike charge ($F\equiv 1$), the photon distribution function can be calculated analytically. We obtain for the Coulomb potential and the modified potential

$$n(\omega,b) = \begin{cases} \dfrac{Z^2\alpha}{\pi^2}\dfrac{\omega}{\gamma^2}\left[K_1\left(\dfrac{\omega b}{\gamma}\right)\right]^2 & \text{retarded Coulomb potential} \\ \\ \dfrac{Z^2\alpha}{\pi^2}\dfrac{\omega}{\gamma^2}\left[\dfrac{K_{1+iZ\alpha}(\omega b/\gamma)}{\Gamma(1-iZ\alpha)}\right]^2 & \text{modified potential (6)} \end{cases} \tag{19}$$

K_ν is a modified Bessel function. For small arguments of the Bessel function one can use the asymptotic expression [22]

$$K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(\frac{1}{2}z)^{-\nu} \tag{20}$$

Therefore, for $\omega b \ll \gamma$ and assumed point-like charges the photon distribution functions (19) nearly completely agree.

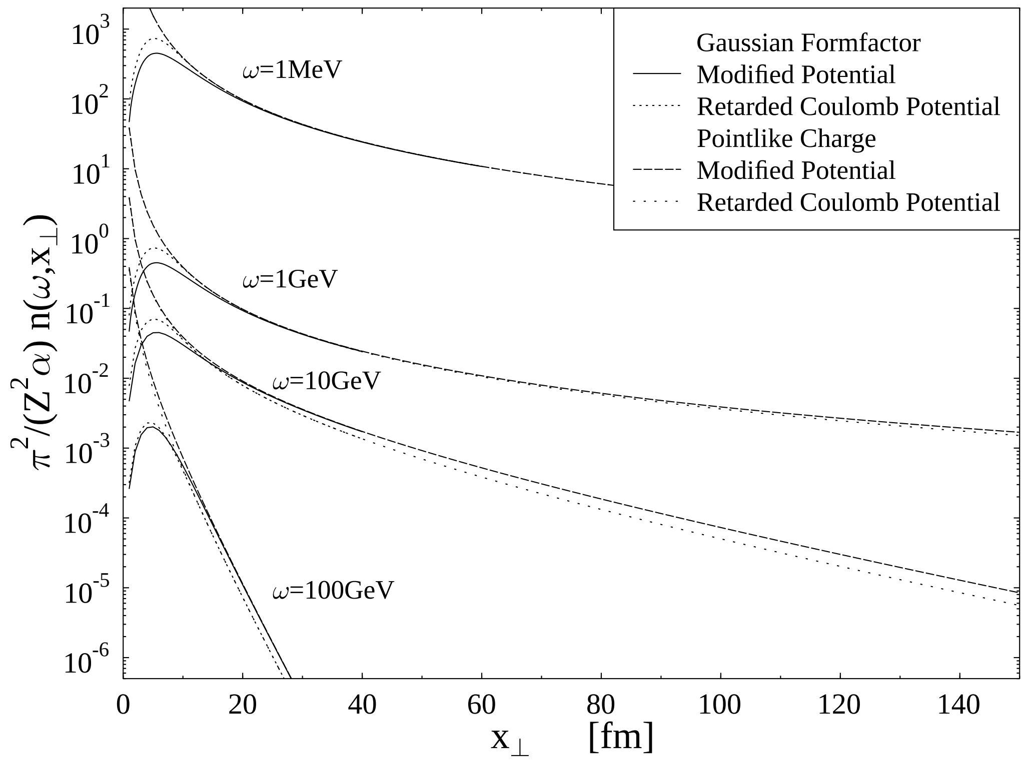

Figure 2: The single-photon distribution function for various photon energies (indicated in the plot) as a function of the transverse distance from the ultrarelativistic charge. The calculations are done for lead ions ($Z = 82$) at LHC energies ($\gamma \approx 3000$).

We have numerically evaluated the photon distribution function (18) for an extended nucleus, using a gaussian form factor $F(Q^2) = e^{-Q^2/(2Q_0^2)}$ with $Q_0 = 60MeV$ which describes the Pb nucleus [23]. Figure 2 shows a comparison between the photon distribution functions for both, point like nuclei ($F \equiv 1$) and extended nuclei usin either the retarded Coulomb potential or the modified potential (6), which represents the exact calculation. At small distances up to a few multiples of the nuclear radius, the modified potential gives a smaller number of equivalent photons than the pure Coulomb potential. For large distances or large photon energies (20) looses its validity and the modified potential gives a larger number of photons than the Coulomb potential (see Figure 3).

The significance of these results can be assessed in single-photon induced processes, such as the electromagnetic dissociation of nuclei in peripheral heavy-ion collisions. In the Weizsäcker-Williams approach the dissociation probability at a given impact parameter is calculated by integrating the product of the measured photon-nucleus dissociation cross section of the target, $\sigma_T^\gamma(\omega)$, and the equivalent photon distribution, $n_P(\omega, b)$, of the

projectile (18) over the photon energy [24, 25]

$$P(b) = \int d\omega\, n_P(\omega, b)\, \sigma_T^{\gamma}(\omega)$$

As shown in [24] for $Pb + Pb$ collisions at LHC energies, the dissociation probability violates unitarity at small impact parameters up to 25 fm. At small impact parameters, however, the photon distribution of the modified potential is suppressed relative to that of the pure Coulomb potential and yields a reduction of the probabiltiy, whereas at large impact parameters, the dissociation probability is enhanced. The reduction visible in Figure 2 taken alone is too small to cure the unitarity violation. Possibly this can be achieved if in addition multi-phonon excitations [27, 26] are taken into account, which also reduce the dissociation probability at small impact parameters.

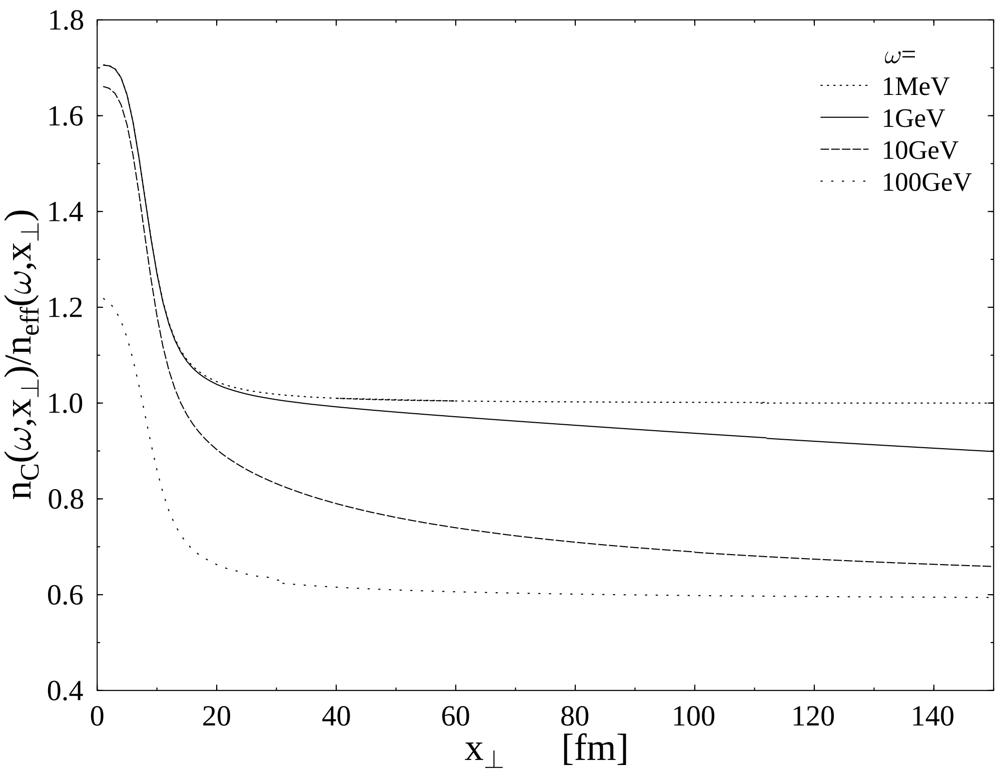

Figure 3: The ratio of the equivalent photon numbers of the pure Coulomb potential, $n_C(\omega, x_\perp)$, and the photon numbers of the effective potential, $n_{eff}(\omega, x_\perp)$. At small transverse distances, one finds a deviation of up to 70% for small photon energies. Far outside the nucleus, the photon distribution functions are determined by (19). For large distances one asymptotically finds a deviation in the order of 40%, independent of the photon energy.

The impact parameter dependent two-photon distribution functions also have to be

corrected according to (18), thus correcting the pair production probability (15), i.e. (8), for Coulomb effects. Due to the complicated convolution of the single-photon distribution functions in (16) and (17), effects on the two-photon distribution function are not obvious. For large distances $x_\perp \gg b$, however, one directly finds an enhancement of the equivalent two-photon numbers.

5 Summary

It was shown by either summing the perturbation series [1] or by matching plane waves at the delta function potential on the light front [4, 12, 1], that the eikonal expression for the scattering amplitude becomes exact in the ultrarelativistic limit ($\gamma \to \infty$). This allows to neglect the squared momentum transfer k^2 relative to the term $2p_i \cdot k$ in the denominator of the propagator of the scattered particle. As a consequence the applicability conditions of the Weizsäcker-Williams method are fulfilled automatically – irrespective of the mass of the scattered particle.

Furthermore, the exact validity of the eikonal formula for ultrarelativistic scattering processes confirm the Pomeranchuk theorem, stating that the cross sections for antiparticle and particle scattering at a given target become identical in the ultrarelativistic limit. In analogy to pomeron exchange in hadron physics, on can describe the exact interaction as the exchange of an effective photon, according to a modified, effective potential given by (6). The cross section, as a peculiarity of the Coulomb interaction, becomes identical to the Mott result. The exact pair production cross section in the field of two ultrarelativistic colliding (pointlike) ions also reduces to the second order perturbative result [12] which was evaluated in [14]. This allows for two conclusions: i) The production rate scales with the square of the target and the projectile charge [9, 10] ii) Asymmetries in the electron and positron spectra should not occur.

Note, however, that the presented formalism is valid only if the produced particles are fas with respect to both nuclei. Therefore, the observed [9] asymmetry at small electron and positron momenta remains unaffected by these considerations.

We applied the Weizsäcker-Williams approach to pair production using the modified potential (6), correctly accounting for the Coulomb boundary conditions. The impact parameter dependent single-photon distribution calculated with the modified potential shows deviations from the equivalent photon distribution function obtained from the retarded Coulomb potential in order of up to 70% at small separations and approximately 40% at large separations from the ion.

In combination with multi-phonon excitation, the correct treatment of Coulomb distortions possibly solves the problem of unitarity violation in photodissociation processes in ultrarelativistic heavy-ion collisions.

The pair production probability is also subject to changes due to the modified photon numbers at given impact parameters and photon energies. The perturbation theoretical probability, as calculated here, rather represents the average number of produced pairs and exceeds unity at sufficiently small impact parameters. The "true" pair production probability has to be corrected by the vacuum-to-vacuum amplitude, which in turn can be calculated from the perturbative pair production probability [28, 29]. This nontrivial influence on the pair production cross section is subject of further studies.

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